



**pyecsca**  
*[pietska]*

## Reverse engineering black-box elliptic curve cryptography via side-channel analysis

Jan Jancar<sup>1</sup>, Vojtech Suchanek<sup>1</sup>, Petr Svenda<sup>1</sup>, Vladimir Sedlacek<sup>2</sup>, Łukasz Chmielewski<sup>1</sup>



CHES 2024  
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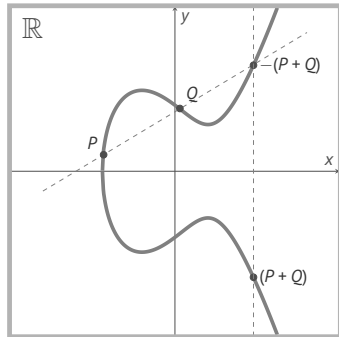
# Outline

- Why?
  - Elliptic Curve Cryptography
  - Side-Channel Attacks
- RQ1: Real-world ECC implementations
- RQ2: Space of possible ECC implementations
- RQ3: Reverse-engineering ECC implementations
- Conclusions

# Why?

## Elliptic Curve Cryptography

- **Elliptic Curve:**  $y^2 \equiv x^3 + ax + b$ 
  - Points  $(x, y) \in E(\mathbb{K})$  form an abelian group
  - Scalar multiplication
$$[n] : E(\mathbb{K}) \rightarrow E(\mathbb{K})$$
$$P \mapsto [n]P = \underbrace{P + P + \dots + P}_{n \text{ times}}$$
  - ECDLP: Find  $x$  given  $[x]G$  and  $G \in E(\mathbb{F}_p)$   
Generally hard when  $\mathbb{K} = \mathbb{F}_p$



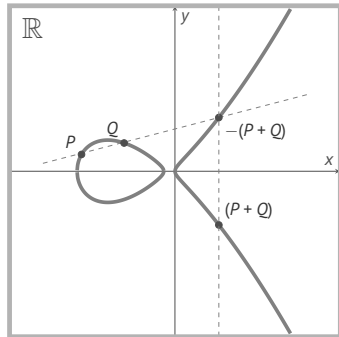
Short Weierstrass



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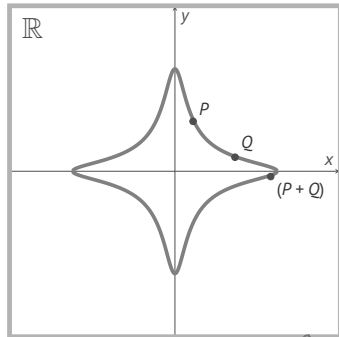
Montgomery



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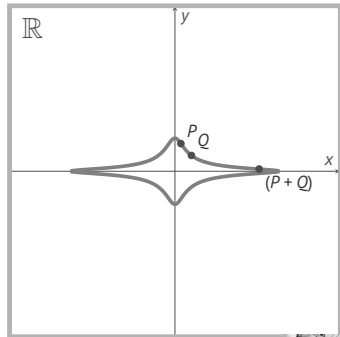
Edwards



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Twisted Edwards



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### ■ XDH, EdDSA, ...

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    - ▶  $(X, Y, Z)$
    - ▶  $(X, Y, Z, ZZ) \dots$

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```
Y1Z2 = Y1*Z2
X1Z2 = X1*Z2
Z1Z2 = Z1*Z2
u = Y2*Z1-Y1Z2
uu = u2
v = X2*Z1-X1Z2
vv = v2
vvv = v*vv
R = vv*X1Z2
A = uu*Z1Z2-vvv-2*R
X3 = v*A
Y3 = u*(R-A)-vvv*Y1Z2
Z3 = vvv*Z1Z2
```

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Y1Z2 = Y1*Z2
U1 = X1*Z2
U2 = X2*Z1
S1 = Y1*Z2
S2 = Y2*Z1
ZZ = Z1*Z2
T = U1+U2
M = S1+S2
R = T2-U1*U2+a*ZZ2
F = ZZ*M
L = M*F
G = T*L
W = R2-G
X3 = 2*F*W
Y3 = R*(G-2*W)-L2
Z3 = 2*F*F2
```

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$$Y1Z2 = Y1 * Z2$$

$$U1 = X1 * Z2$$

$$u = Y2 * Z1 - Y1 * Z2$$

$$v = X2 * Z1 - X1 * Z2$$

$$A = u^2 * Z1 * Z2 - v^3 - 2 * v^2 * X1 * Z2$$

$$X3 = v * A$$

$$Y3 = u * (v^2 * X1 * Z2 - A) - v^3 * Y1 * Z2$$

$$Z3 = v^3 * Z1 * Z2$$

$$R = T^2 - U1 * U2 + a * Z2^2$$

$$F = Z2 * M$$

$$L = M * F$$

$$G = T * L$$

$$W = R^2 - G$$

$$X3 = 2 * F * W$$

$$Y3 = R * (G - 2 * W) - L^2$$

$$Z3 = 2 * F * F^2$$

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## Elliptic Curve Cryptography

- Many implementation possibilities
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  - Scalar multiplier
    - ▶ *fixed-base, variable-base, multi-scalar*

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---

### Algorithm Left-to-right double-and-add

---

**function** LTR( $G, k = (k_l, \dots, k_0)_2$ )

$R = \mathcal{O}$

**for**  $i = l$  **downto** 0 **do**

$R = \text{dbl}(R)$

**if**  $k_i = 1$  **then**

$R = \text{add}(R, G)$

**return**  $R$

---



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---

### Algorithm Fixed-window scalar multiplier

---

```
function Window( $G, k = (k_l, \dots, k_0)_2$ )  
   $PrecomputedTable = [0 * G, 1 * G, \dots, 2^w - 1 * G]$   
   $\hat{k} = \text{recode } k \text{ to } w\text{-bit windows}$   
   $T = \mathcal{O}$   
  for  $i = 1$  to  $|\hat{k}|$  do  
     $T = 2^w T$   
     $T = T + PrecomputedTable[\hat{k}_i]$   
  return  $T$ 
```

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## Elliptic Curve Cryptography

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    - ▶ Inversion: *GCD, Euler*

# Why?

## Side-Channel Attacks

- Simple Power Analysis
- Differential Power Analysis
- Correlation Power Analysis
- Mutual Information Analysis
- Refined Power Analysis, Zero-value Point Attack, Exceptional Procedure Attack
- Template attacks
- Leakage assessment
- Doubling attack, Collision attacks
- ...

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# Why?

## Assumptions

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$\mathbb{F}_p$  with  $p \neq \{2, 3\}$ . The algorithm used for the hardware modular multiplication is assumed to be known to the attacker. Moreover, to simplify the attack

---

<sup>1</sup> Aurélie Bauer, Eliane Jaulmes, Emmanuel Prouff, Jean-René Reinhard & Justine Wild: Horizontal Collision Correlation Attack on Elliptic Curves

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## Assumptions

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## Assumptions

input to  $s$  (“fix class”). (This assumes a white-box evaluator that has access to implementation internals.)

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## Assumptions

assumes a white-box evaluator

e 6.1 abstractly depicts a side-channel measurement of such an exponent. For the sake of simplicity, I assume it is a binary exponent

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---

<sup>3</sup> Johann Heyszl: Impact of Localized Electromagnetic Field Measurements on Implementations of Asymmetric Cryptography

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values may be manipulated when working with points  $P$  and  $2P$ . However this idea only works when using the downward routine.

---

<sup>4</sup> Pierre-Alain Fouque & Frederic Valette: The Doubling Attack – Why Upwards Is Better than Downwards

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## Assumptions

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a doubling operation from an addition one. This technique, which allows to eventually recover the secret scalar, is applied to three different atomic formulae on elliptic curves,

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attack on the Montgomery-López-Dahab ladder algorithm

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<sup>5</sup> Bo-Yeon Sim & Dong-Guk Han: Key Bit-Dependent Attack on Protected PKC Using a Single Trace

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full knowledge of all algorithms,

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<sup>6</sup> Jean-Luc Danger, Sylvain Guilley, Philippe Hoogvorst, Cédric Murdica & David Naccache: A synthesis of side-channel attacks on elliptic curve cryptography in smart-cards

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knowledge of the ECSM and the elliptic

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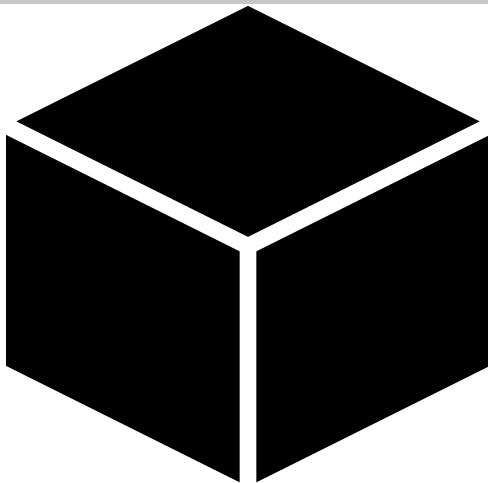
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**Lots of assumptions you've got there!**

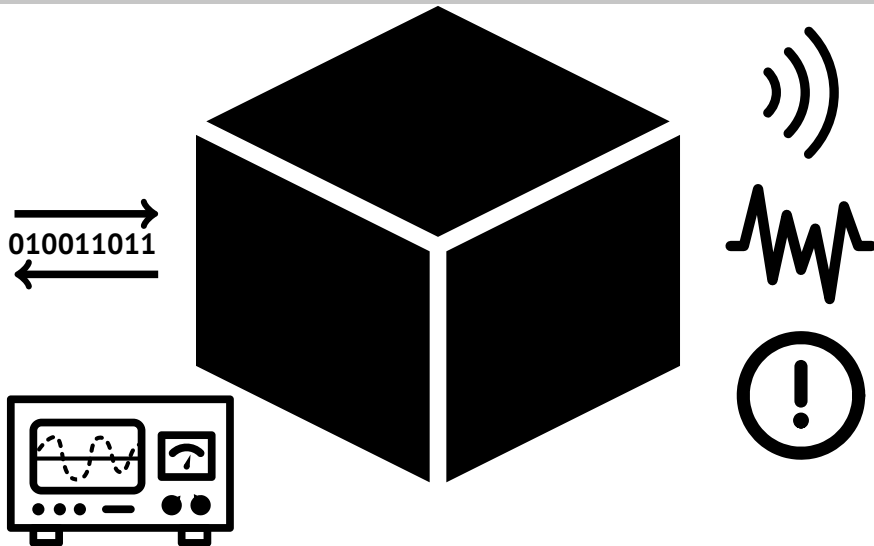
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Reality



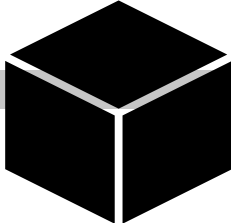
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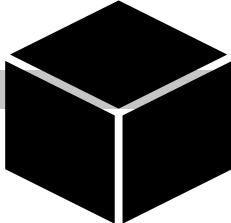
## Reality



- Real-world cryptographic hardware is usually a black-box
  - TPMs, HSMs, smartcards, ...
- Why?
  - Security by obscurity
  - Certifications encourage information hiding (CC, JIL-SCA)
- Contrast to cryptographic theory space
  - Kerckhoffs's principle
  - Open design, open discussion

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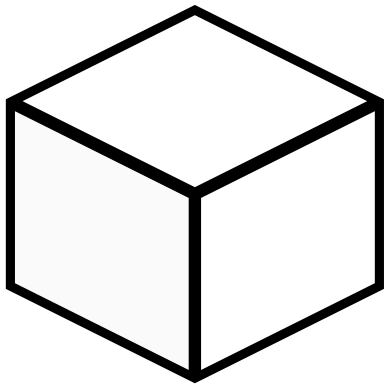
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Auguste Kerckhoffs

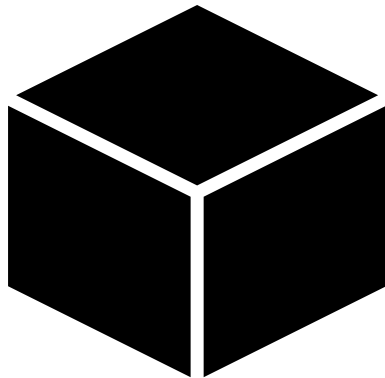
# Why?

Gap



**Attacks**

???



**Targets**

**RQ1:**

What implementation choices are used in real-world open-source ECC libraries?

**RQ2:**


How large is the space of all possible ECC implementations?

**RQ3:**

Is it possible to automatically reverse-engineer black-box ECC implementations?

# RQ1


## What implementation choices are used in real-world open-source ECC libraries?

- Analyzed 18 open-source ECC libraries (  )
- *BearSSL, BoringSSL, Botan, BouncyCastle, fastecdsa, Go crypto, Intel IPP cryptography, libgcrypt, LibreSSL, libsecp256k1, libtomcrypt, mbedTLS, micro-ecc, Nettle, NSS, OpenSSL, SunEC, and Microsoft SymCrypt*




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- Full report: <https://pyecdsa.org/libraries.html>

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## What implementation choices are used in real-world open-source ECC libraries?

- **Specific implementations**
  - Curve or architecture-based (10 📄 )
  - e.g.  $a = -3$  or special prime arithmetic

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- Usually outside = inside
- Montgomery outside, Twisted-Edwards inside (4 📄 )


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- *fixed-base + variable-base + multi-scalar*
- Comb, fixed-window, wNAF, GLV, ...
- 4 to 7 bit widths


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### ■ Addition formulas

- 112 formula implementations
- 50 “standard” (EFD)
- 23 out-of-scope
- 39 “non-standard”
- Expanded standard formulas from ~200 to ~20000



# RQ1

## What implementation choices are used in real-world open-source ECC libraries?

### ■ Specific implementations

- Curve or architecture-based (10 📄)
- e.g.  $a = -3$  or special prime arithmetic

### ■ Curve models

- Usually outside = inside

**Wide range of implementation choices in real-world implementations.**

Twisted-Edwards inside (4 📄)



### ■ Scalar multiplier

- *fixed-base + variable-base + multi-scalar*
- Comb, fixed-window, wNAF, GLV, ...
- 4 to 7 bit widths

### ■ Coordinate systems

- Usually Jacobian, also homogenous or xz

### ■ Addition formulas

- 112 formula implementations
- 50 “standard” (EFD)
- 39 “non-standard”

From standard formulas from ~200 to ~20000

# RQ2

## How large is the space of all possible ECC implementations?

- We can enumerate:
  - Curve model
  - Coordinate system
  - Addition formulas
  - Scalar multiplier
  - Misc. options

# RQ2

## How large is the space of all possible ECC implementations?

- We can enumerate:
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  - Misc. options
- Total: 139 489

Curve	Coords	#	Total
$\mathcal{E}_{\text{SW}}$	jacobian	17 136	113 502
	jacobian-0	22 848	
	jacobian-3	28 560	
	modified	2 856	
	projective	9 520	
	projective-1	10 710	
	projective-3	16 660	
	w12-0	476	
	xyzz	1 428	
	xyzz-3	2 856	
	xz	452	
$\mathcal{E}_{\text{M}}$	xz	132	132
$\mathcal{E}_{\text{E}}$	inverted	2 856	14 431
	projective	11 424	
	yz	99	
	yzsquared	52	
$\mathcal{E}_{\text{TE}}$	extended	2 856	11 424
	extended-1	5 712	
	inverted	1 428	
	projective	1 428	

# RQ2

## How large is the space of all possible ECC implementations?

- We can enumerate:
  - Curve model
  - Coordinate system
  - Addition formulas
  - Scalar multiplier
  - Misc. options
- Total: 139 489

Scalar multiplier	#
LTR	9 328
RTL	9 328
Coron	1 166
Ladder	407
SimpleLadder	2 332
DiffLadder	328
BinaryNAF	4 664
WindowNAF	18 656
WindowBooth	18 656
Window	9 328
SlidingWindow	18 656
FullPrecomp	18 656
Comb	9 328
BGMW	18 656

# RQ2

## How large is the space of all possible ECC implementations?

- We can enumerate:

- Curve model
- Coordinate system
- Addition formulas

**Considerable number of implementation configurations: 139 489.**

- Misc. options

- Total: 139 489



**Worth reverse engineering.**

# RQ3

## Is it possible to automatically reverse-engineer black-box ECC implementations?

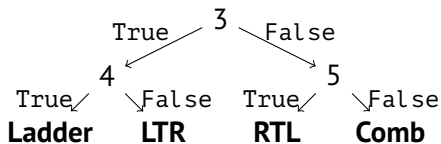
- **Idea:** Use side-channel attacks and turn them around
  - ~~Assume knowledge of the impl. and target the key~~
  - Assume knowledge of the key and target the impl.
- Concretely “*special-point-based*” attacks: **RPA, ZVP, EPA**
- Can recognize when a special point appears in scalar multiplication
- **Idea:** Behavior of different implementations differs under these attacks

# RQ3

## Is it possible to automatically reverse-engineer black-box ECC implementations?

- Simulate behavior of implementations under the oracle (attack)
  - **RPA**: Is  $[r]P$  computed during  $[k]P$  computation by the target?
- Build a decision table with the answers
- Build a decision tree, recursively picking the best split

$\mathcal{I}_{\text{RPA}}$ :	$[2^{-1}]P_0$	$[3^{-1}]P_0$	$[4^{-1}]P_0$	$[5^{-1}]P_0$
<b>LTR</b>	True	True	False	False
<b>RTL</b>	True	False	True	True
<b>Comb</b>	True	False	True	False
<b>Ladder</b>	True	True	True	False
...	...			



# RQ3

## Is it possible to automatically reverse-engineer black-box ECC implementations?

- Simulate behavior of implementations under the oracle (attack)
  - **RPA**: Is  $[r]P$  computed during  $[k]P$  computation by the target?
- Build a decision table with the answers
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Method	Curve	Coordinates	Formulas	Multiplier	Scalar	Input point
RPA-RE	chosen	any	any	<b>target</b>	known	chosen
ZVP-RE	chosen	<b>target</b>	<b>target</b>	known	known	chosen
EPA-RE	chosen	<b>target</b>	<b>target</b>	known	known	chosen



# RQ3

## Is it possible to automatically reverse-engineer black-box ECC implementations?








- Implemented in the **pyecsca** toolkit
- It works!

Method	Oracle	$ \mathcal{C} $	# 	Expected		Random	
					 		 
RPA-RE	binary	34	34	1.0	5.0	1.0	5.0
ZVP-RE	binary	214	74	8.7	5.1	5.0	4.0
ZVP-RE	count	214	134	2.4	4.0	1.3	2.5
ZVP-RE	position	214	196	1.2	2.1	1.1	1.8

# RQ3

## Is it possible to automatically reverse-engineer black-box ECC implementations?

- Implemented in the **pyecsca** toolkit
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**Yes, it is possible.**

...and much more



**pyecsca**

[pyecsca.org](https://pyecsca.org)

### **Python Elliptic Curve Side Channel Analysis toolkit**

- Can enumerate configurations
- Can simulate computation given any configuration
- Can generate C implementations of ECC for micro-processors
- Can perform power and EM-tracing
- Can process collected traces and visualize them
- Can perform known attacks against ECC
- **Can be used to reverse engineer ECC**

# Conclusions

- Documented large variety of implementation choices in 18 open-source ECC libraries
  - **Expect similar variety in black-box devices**
- Explored the space of possible implementation choices of ECC
  - **Considerable number of choices, necessary knowledge for an attack**
- Presented several novel attack-based reverse-engineering methods for ECC
  - **Demonstrated effectiveness on two simulation levels**
- Explore our tutorial:

[github.com/J08nY/pyecsca-tutorial-ches2024](https://github.com/J08nY/pyecsca-tutorial-ches2024)



**pyecsca**  
*[pietska]*

## Reverse engineering black-box elliptic curve cryptography via side-channel analysis

Jan Jancar, Vojtech Suchanek, Petr Svenda, Vladimir Sedlacek, Łukasz Chmielewski

Questions?

 J08nY

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# References

- Jan Jancar, Vojtech Suchanek, Petr Svenda, Vladimir Sedlacek & Łukasz Chmielewski;  
[pyecsca: Reverse-engineering black-box elliptic curve cryptography via side-channel analysis](#)

# References (cont.)

## Attack assumptions

- 1  Aurélie Bauer, Eliane Jaulmes, Emmanuel Prouff, Jean-René Reinhard & Justine Wild: [Horizontal Collision Correlation Attack on Elliptic Curves](#)
- 2  Oscar Reparaz, Josep Balasch & Ingrid Verbauwhed: [Dude, is my code constant time?](#)
- 3  Johann Heyszl: [Impact of Localized Electromagnetic Field Measurements on Implementations of Asymmetric Cryptography](#)
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- 5  Bo-Yeon Sim & Dong-Guk Han: [Key Bit-Dependent Attack on Protected PKC Using a Single Trace](#)
- 6  Jean-Luc Danger, Sylvain Guilley, Philippe Hoogvorst, Cédric Murdica & David Naccache: [A synthesis of side-channel attacks on elliptic curve cryptography in smart-cards](#)

## Other

 <https://hyperelliptic.org/EFD/>

 Icons from  **Noun Project** &  **Font Awesome**

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## ECC attack and countermeasure surveys

- Junfeng Fan, Xu Guo, Elke De Mulder, Patrick Schaumont, Bart Preneel & Ingrid Verbauwhede; [State-of-the-art of secure ECC implementations: A survey on known side-channel attacks and countermeasures](#)
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- Rodrigo Abarzúa, Claudio Valencia Cordero & Julio Cesar López-Hernández; [Survey on performance and security problems of countermeasures for passive side-channel attacks on ECC](#)



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## Special-point-based attacks

- Louis Goubin; [A Refined Power-Analysis Attack on Elliptic Curve Cryptosystems](#)
- Toru Akishita, Tsuyoshi Takagi; [Zero-value point attacks on elliptic curve cryptosystem](#)
- Tetsuya Izu, Tsuyoshi Takagi; [Exceptional procedure attack on elliptic curve cryptosystems](#)
- Vladimir Sedlacek, Jesús-Javier Chi-Domínguez, Jan Jancar, Billy Bob Brumley;  
[A formula for disaster: a unified approach to elliptic curve special-point-based attacks](#)

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- Jean-Jacques Quisquater & David Samyde;  
[Automatic code recognition for smart cards using a Kohonen neural network](#)
- Dennis Vermoen, Marc F. Witteman & Georgi Gaydadjiev;  
[Reverse engineering Java Card applets using power analysis](#)
- Thomas Eisenbarth, Christof Paar & Björn Weghenkel;  
[Building a side channel based disassembler](#)
- ...and much more (see the paper)

# References (cont.)

## Side-channel-based reverse engineering

- Christophe Clavier;  
[Side channel analysis for reverse engineering \(SCARE\) – an improved attack against a secret A3/A8 GSM algorithm](#)
- Rémy Daudigny, Hervé Ledig, Frédéric Muller & Frédéric Valette;  
[SCARE of the DES](#)
- Manuel San Pedro, Mate Soos & Sylvain Guilley;  
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- Frederic Amiel, Benoit Feix & Karine Villegas;  
[Power analysis for secret recovering and reverse engineering of public key algorithms](#)
- ...and some more (see the paper)

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## Manual reverse engineering

- Thomas Roche, Victor Lomné, Camille Mutschler & Laurent Imbert;  
[A Side Journey to Titan](#)
- Thomas Roche;  
[EUCLEAK: Side-Channel Attack on the YubiKey 5 Series](#)